Our new definitions of sine and cosine give values for any angle α. But this is not quite true for our new definitions of tangent, cotangent, secant and cosecant, because they involve division. We must be sure that we are not dividing by 0.

Indeed, we will not define $\tan\alpha$ if $\cos\alpha = 0$. Expressions such as $\tan 90°$, $\tan 270°$, and $\tan(-90°)$ must remain undefined.

For similar reasons, we cannot define $\cot 0°$, or $\csc 180°$.

Exercises

1. Find the numerical value of the following expressions. Do this without using your calculator, then check your answers with your calculator.

   a) $\sin 390°$ b) $\cos 3720°$ c) $\tan 1845°$

   d) $\sin 315°$ e) $\cot 420°$ f) $\tan(-30°)$

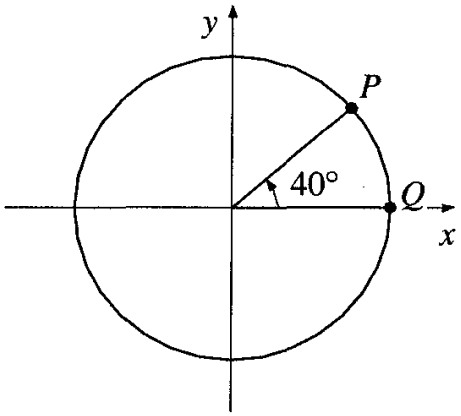
2. Find the numerical value of each expression below, or indicate if the given expression is undefined.

   a) $\tan 360°$ b) $\sin 180°$ c) $\cos 180°$

   d) $\cot 90°$ e) $\cot 360°$ f) $\tan(-270°)$

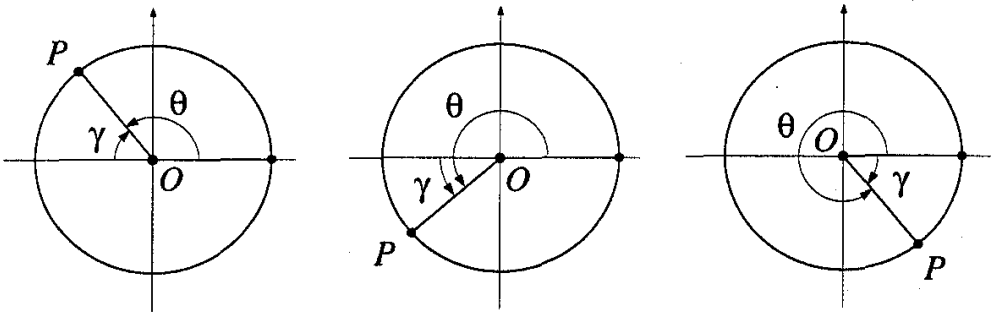
4 Calculations with angles of rotations

Let us look back at our original picture of an angle in a circle:



Originally, we thought of this as an angle of 40°. But a diagram of a 400° angle would look exactly the same, as would a diagram for 760° or −320°.

made by one side of the angle and the x-axis (the angle marked γ in each diagram below):



Example 35 Find $\sin 300°$.

Solution. Point P, having rotated through 300°, will end up in quadrant IV. So $\sin 300°$ is negative. Furthermore, the angle made by one side and the x-axis is 60°. Hence $\sin 300° = -\sin 60° = -\sqrt{3}/2$. □

Exercises

1. In what quadrant will the point P lie after a rotation of 400°? 3600°? 1845°? −30°? −359°?

2. Fill in the table below (you won't need a calculator). What is the relationship between $\sin\alpha$ and $\sin(-\alpha)$?

$\sin 30°$		$\sin(-30°)$	
$\sin 135°$		$\sin(-135°)$	
$\sin 210°$		$\sin(-210°)$	
$\sin 300°$		$\sin(-300°)$	
$\sin 390°$		$\sin(-390°)$	
$\sin 480°$		$\sin(-480°)$	

3. Solve the following equations for α, where $0 < \alpha < 360°$:

a) $\sin\alpha = 0$ b) $\cos\alpha = 0$ c) $\sin\alpha = 1$

d) $\cos\alpha = 1$ e) $\sin\alpha = -1$ f) $\cos\alpha = \frac{1}{2}$

g) $\sin\alpha = -\frac{1}{2}$ h) $\sin^2\alpha = \frac{1}{2}$ i) $\cos^2\alpha = -\frac{3}{4}$

4. a) If $\sin\alpha = 5/13$, in what quadrant can α lie? What are the possible values of $\cos\alpha$?

Exercises

1. What is the radian measure of an angle of 180°? 90°?
2. What is the degree measure of an angle of 2 radians?
3. What is the radian measure of 1/4 of a full rotation?
4. What is the radian measure of a rotation through an angle of 45°?
5. Fill in the following table:

Degree measure	Radian Measure
90	
180	
270	
360	
	$\pi/2$
	π
	$3\pi/2$
	2π

6. Fill in the following tables:

Degree measure	Radian measure
0	
30	
72	
120	
135	
	$\pi/6$
	$\pi/5$
	$\pi/4$
	$\pi/3$
	$2\pi/3$
	$7\pi/10$

Degree measure	Radian measure
198	
210	
216	
225	
240	
	$11\pi/10$
	$10\pi/9$
	$7\pi/6$
	$6\pi/5$
	$5\pi/4$
	$4\pi/3$

7. What is the radian measure of an angle of 1 degree?

8. Using your calculator, find the sine of an angle of (a) 1 radian; (b) 1 degree.

9. Without using your calculator, fill in the following table:

α (in radian)	$\sin\alpha$	$\cos\alpha$
$\pi/6$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$7\pi/6$		
$5\pi/4$		
$3\pi/2$		
$11\pi/6$		

10. In a circle of radius 1, what is the length of an arc cut off by a central angle of 2 radians? Of 3 radians? Of π radians?

11. In a circle of radius 3, what is the length of an arc cut off by a central angle of 2 radians? Of 3 radians? Of π radians?

12. If $\sin\pi/9 = \cos\alpha$ and α is acute, what is the radian measure of α?

13. If α is an angle between 0 and $\pi/2$ (in radian measure), which is bigger: $\sin\alpha$ or $\cos(\pi/2 - \alpha)$?